Office of the Director of National Intelligence
IARPA
BE THE FUTURE

QUANTUM ENHANCED OPTIMIZATION

IARPA Quantum Enhanced Optimization (QEO) Study

Andrew J. Kerman & William Oliver

MIT Lincoln Laboratory

Matthias Troyer

ETH Zürich

Fundamental Questions Underlying the IARPA QEO Study

- **Under what conditions can engineered quantum fluctuations be harnessed to enhance classical optimization?**
  - **The same solutions faster**
  - **Lower-energy solutions**
  - **A wider range of solutions**

- **Can application problems be mapped to spin Hamiltonians that may satisfy these conditions?**

- **If so, what are the corresponding design requirements for realistic, *application-specific* QA machines?**

Assess theoretical potential and practical plausibility for QA to provide disruptive classical optimization capability

QEO Study Team

- **MIT Lincoln Laboratory: Quantum Information and Integrated Nanosystems group - W.D. Oliver, A.J. Kerman, L. Racz, M. Gouker**
- **MIT EECS Department - T. Orlando and S. Gustavsson**
- **UC Berkeley - J. Clarke and I. Siddiqi**
- **ETH Zürich - M. Troyer**
- **Texas A&M University - H. Katzgraber**
- **NASA QuAIL - E. Rieffel (previously: V. Smelyanskiy)**

IARPA: Karl Roenigk (P.M.), Bryan Jacobs, Krystal Brown

Outline

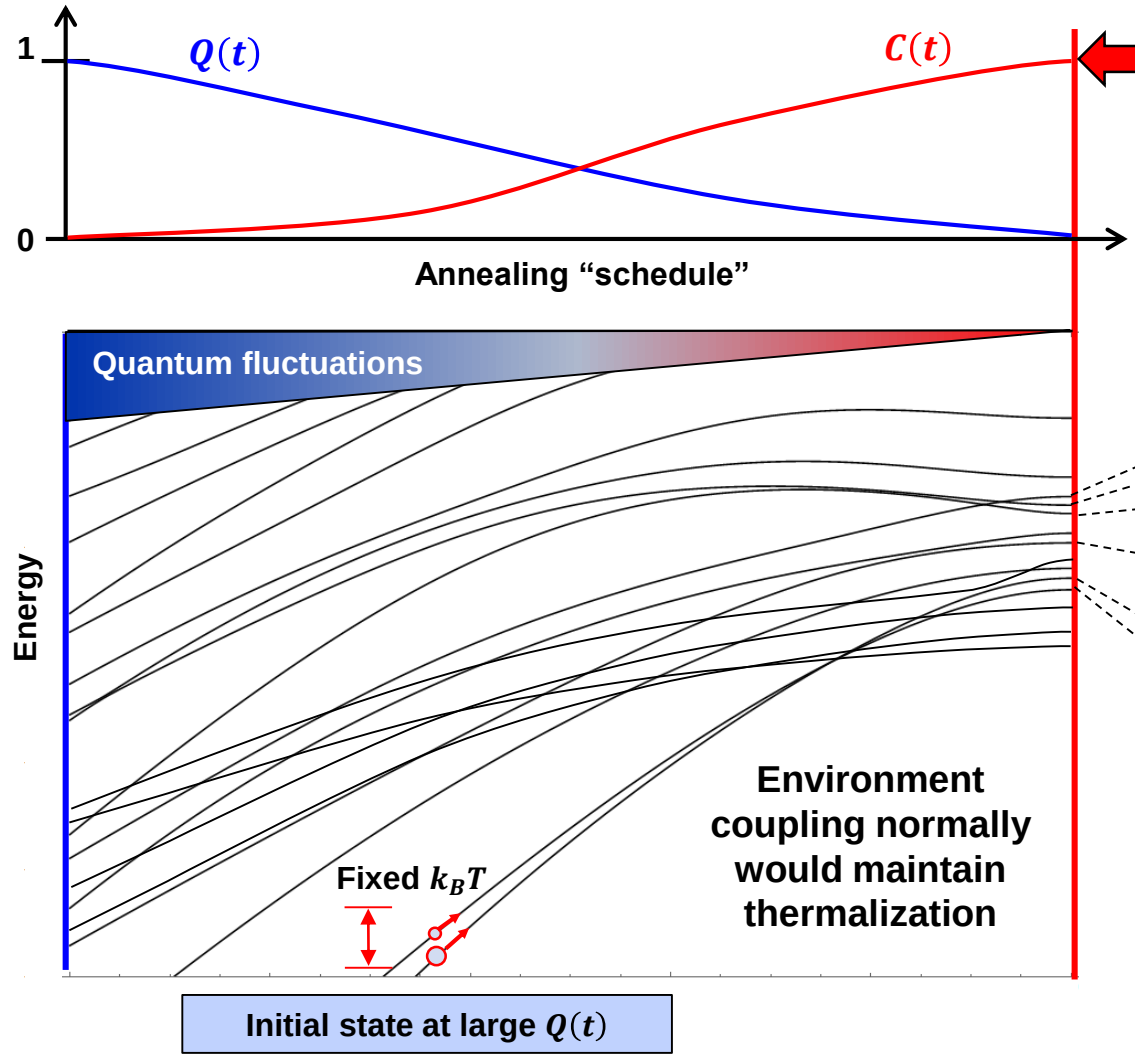
- **Introduction and motivation**
- **Overview of plan developed in QEO study**
- **QEO study concepts and progress**
  - **Physical hardware and architecture concepts: this talk**
  - **Materials, fabrication, and experiment: Will Oliver – next talk**

    Representing also work of MIT and UCB
  - **Theory and simulation: Matthias Troyer – final talk**

    Representing also work of NASA and TAMU

LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Quantum Annealing



Initial state at large $Q(t)$

Classical Ising model

$$H_C = \sum_i Z_i \sigma_i^z + \sum_{i<j} J_{ij} \sigma_i^z \sigma_j^z$$

$$\hat{H}_{\text{tot}} = C(t) H_C + Q(t) \hat{H}_F$$

Quantum "fluctuations:"

$$[\hat{H}_F, \hat{H}_C] \neq 0$$

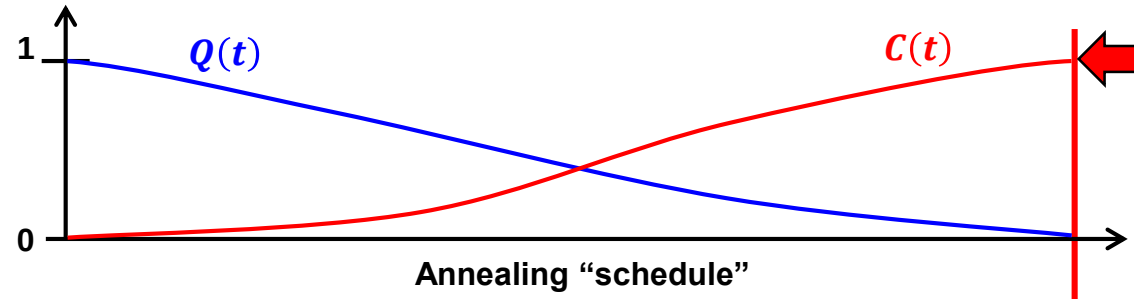
Usual case: $\hat{H}_F = X \sum_i \hat{\sigma}_i^x$

No way to know distance from true ground state

In general, takes exponential time to find it

LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Quantum Annealing



Classical Ising model

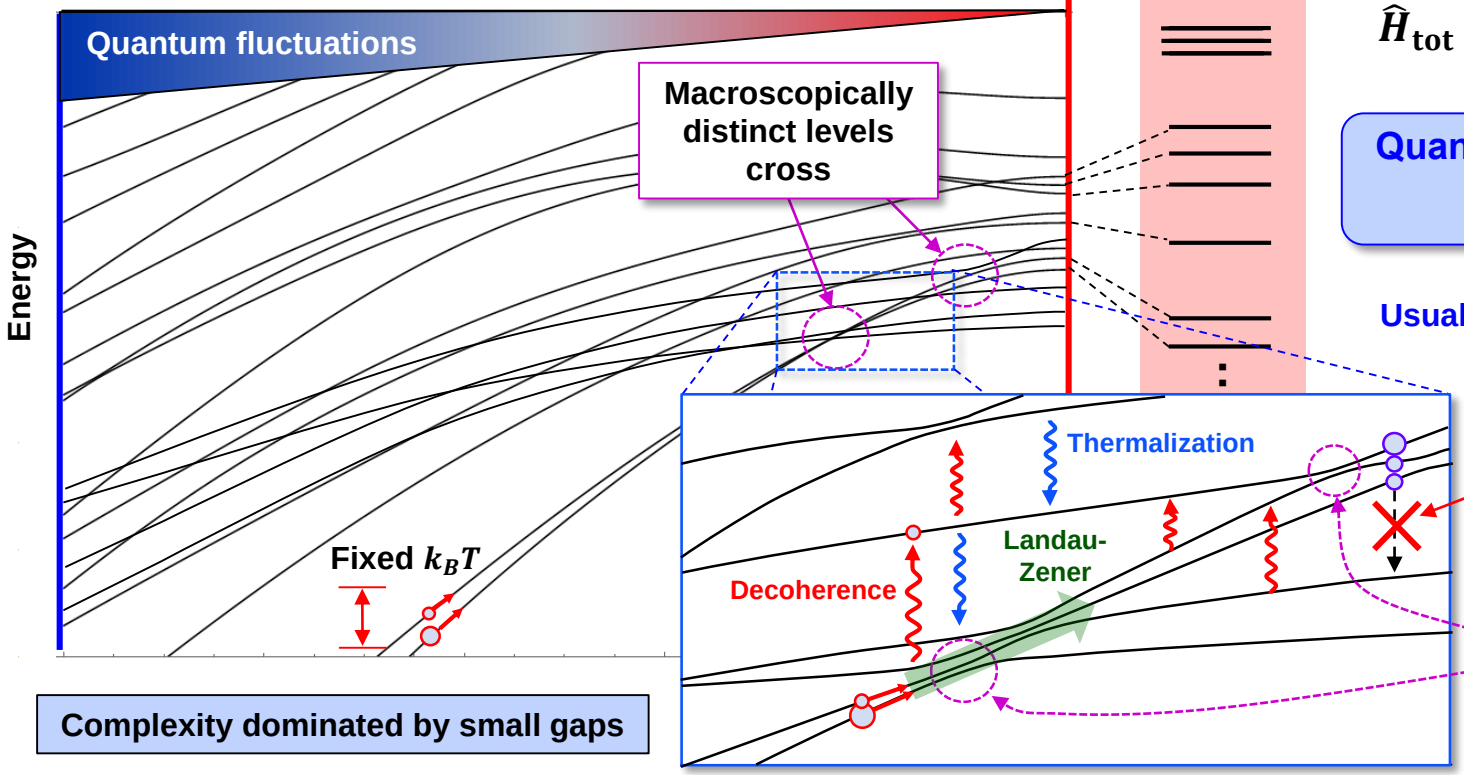
$$H_{\mathcal{C}} = \sum_i Z_i \sigma_i^z + \sum_{i<j} J_{ij} \sigma_i^z \sigma_j^z$$

$$\hat{H}_{\text{tot}} = C(t) H_{\mathcal{C}} + Q(t) \hat{H}_F$$

Quantum "fluctuations:"

$$[\hat{H}_F, \hat{H}_{\mathcal{C}}] \neq 0$$

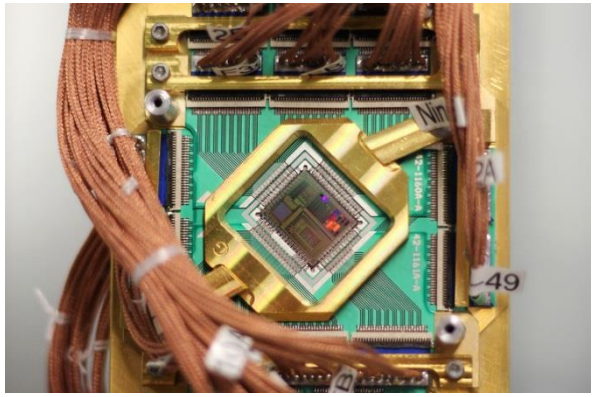
Usual case: $\hat{H}_F = X \sum_i \hat{\sigma}_i^x$



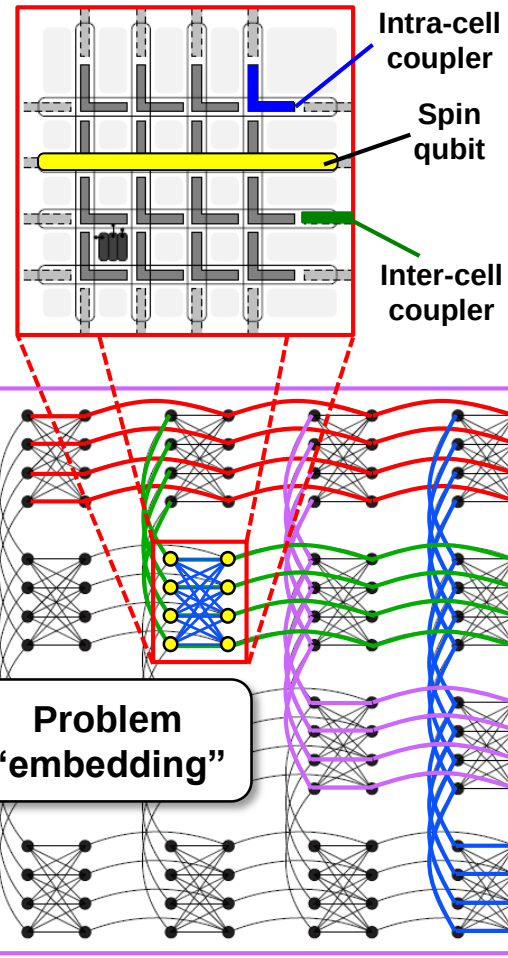
Complexity dominated by small gaps

LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

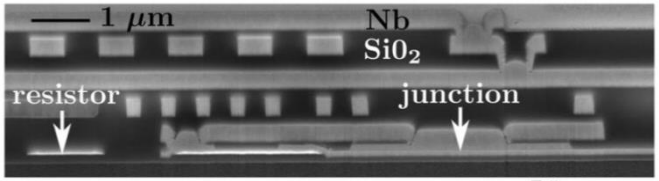
D-Wave Systems QA Machines

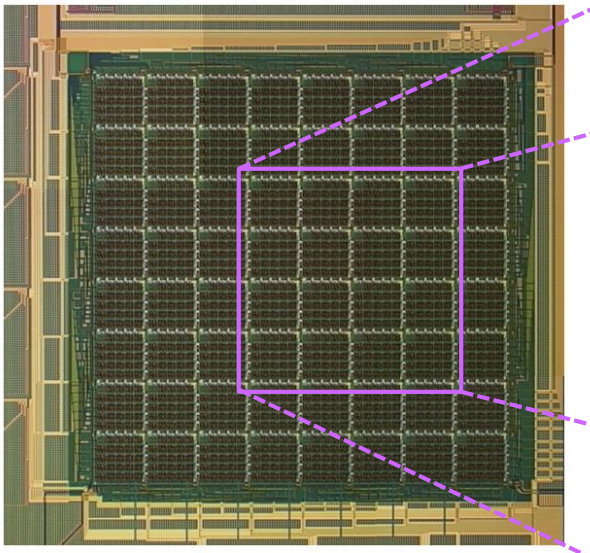


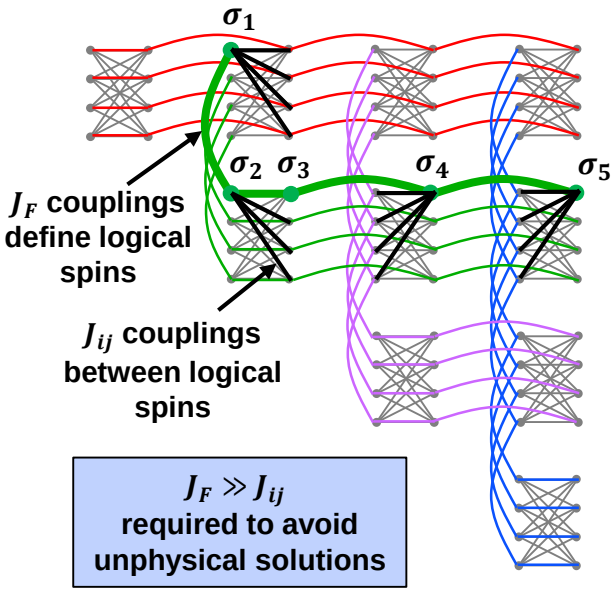
$K_{4,4}$ Chimera graph unit cell



Much of control electronics fully integrated: superconducting SFQ digital logic



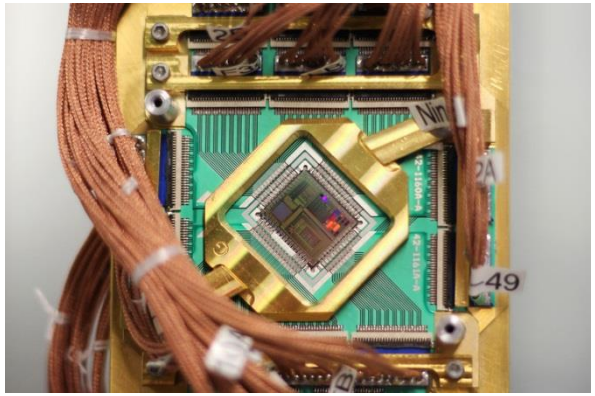




$J_F \gg J_{ij}$ required to avoid unphysical solutions

LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

D-Wave Systems QA Machines



Experience with D-Wave platform suggests:

- **Higher dynamic range/precision in parameter setting**
- **Greater annealing schedule control and bandwidth**
- **Greater Ising connectivity – larger quantum fluctuations in embedded problems**

Logical states of embedded spin:

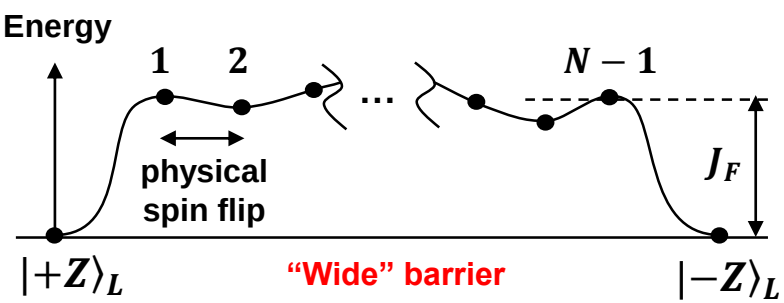
$$|+Z\rangle_L \equiv |+z\rangle_1 \otimes \cdots \otimes |-z\rangle_N$$

$$|-Z\rangle_L \equiv |+z\rangle_1 \otimes \cdots \otimes |-z\rangle_N$$

Logical transverse field requires $N-$ spin Interactions:

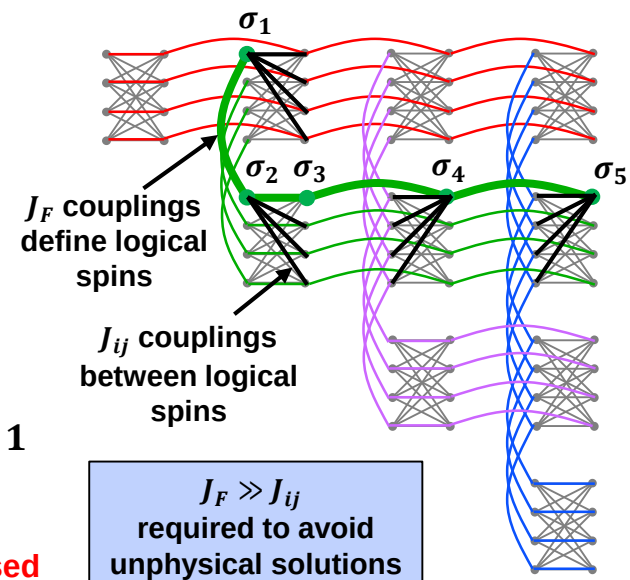
$$\hat{S}_L^x = \prod_{i=1}^{N} \hat{\sigma}_i^x$$

$$\hat{S}_P^x = \sum_{i=1}^{N} \hat{\sigma}_i^x$$



$$\frac{\langle \hat{S}_L^x \rangle}{\langle \hat{S}_P^x \rangle} \sim \left(\frac{\mu B_x}{J_F}\right)^{N-1} \ll 1$$

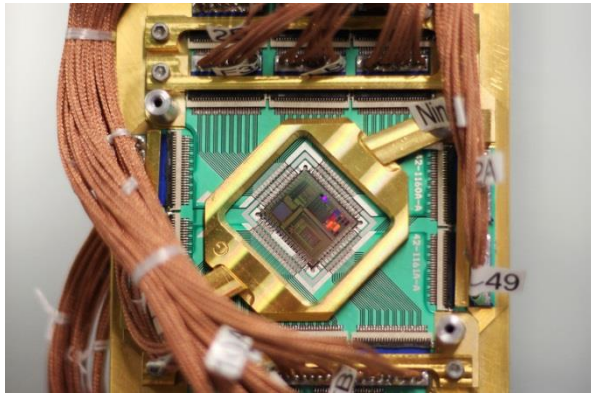
Logical fluctuations exponentially suppressed



$J_F \gg J_{ij}$ **required to avoid unphysical solutions**

LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

D-Wave Systems QA Machines



Experience with D-Wave platform suggests:

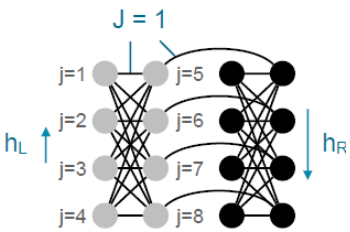
- **Higher dynamic range/precision in parameter setting**
- **Greater annealing schedule control and bandwidth**
- **Greater Ising connectivity – larger quantum fluctuations in embedded problems**

Additional areas of critical interest in QEO

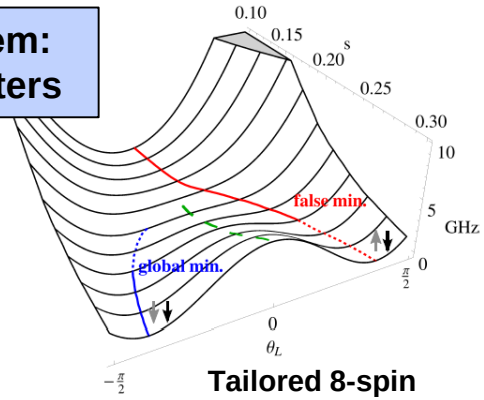
- **Higher spin coherence**
- **Wider range of quantum fluctuations (e.g., multispin, including non-stoquastic) and engineered quantum environments**
- **Real-time monitoring, with potential for adaptive feedback**
- **Quantum error suppression approaches**
- **Application-specific architectures**

Models for the D-Wave Machine

"Tailored" problem: two coupled clusters

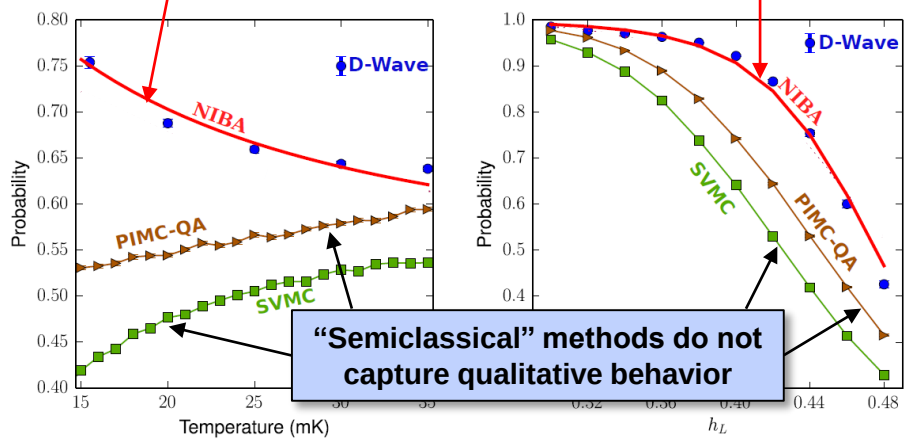


Boixo, *et al.*, arXiv:1502.05754



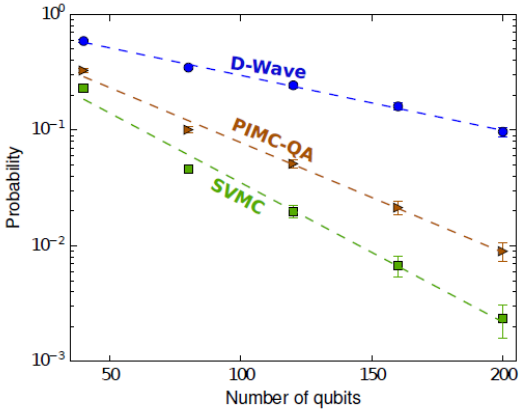
Tailored 8-spin tunnel barrier

Excellent agreement with incoherent tunneling model

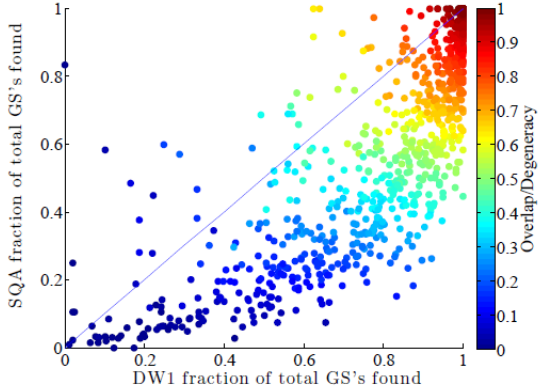


"Semiclassical" methods do not capture qualitative behavior

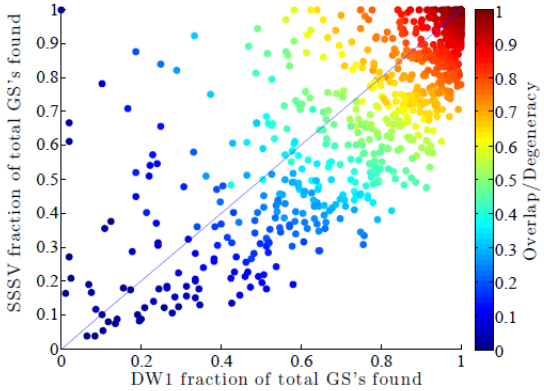
Larger coupled cluster problems



Quantum Monte-Carlo vs. D-Wave



Spin-vector Monte-Carlo vs. D-Wave

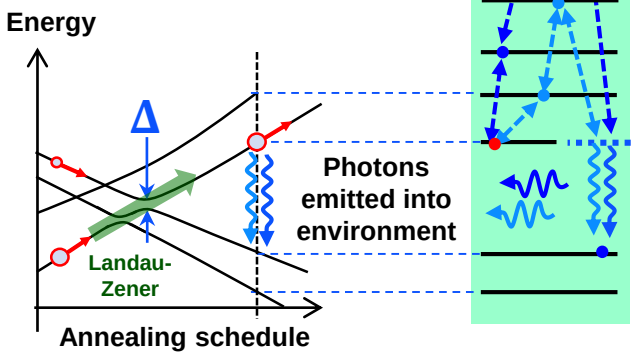


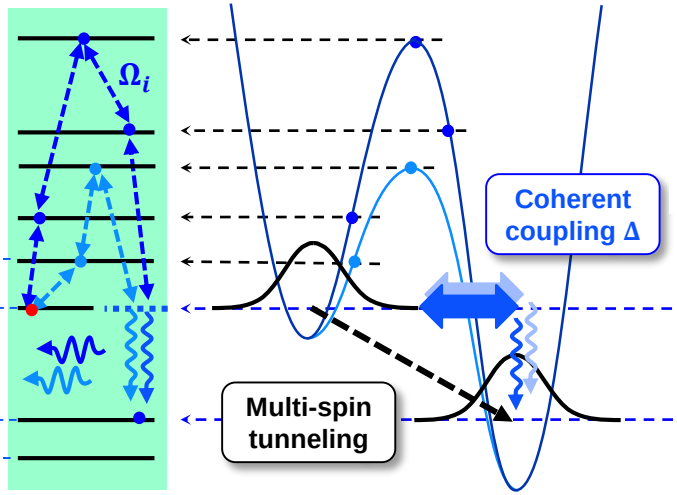
Scalable method for capturing large-scale QA dynamics still lacking

LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

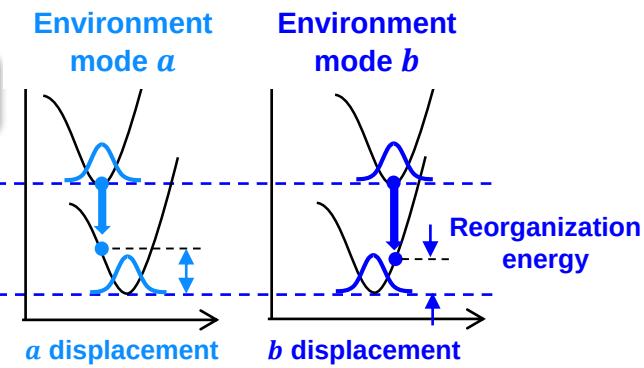
Increasing the "Quantum" in QA

Coupling by driver Hamiltonian

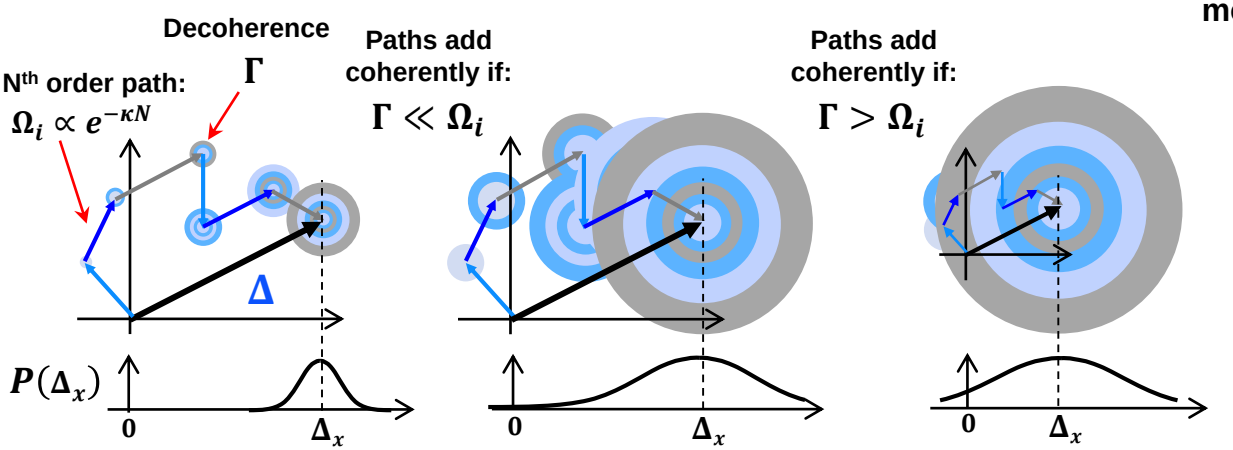




Coupling to spin environment



Each spin emits into separate modes ≡ "incoherent tunneling"



Greater coherence increases effective gap, however:

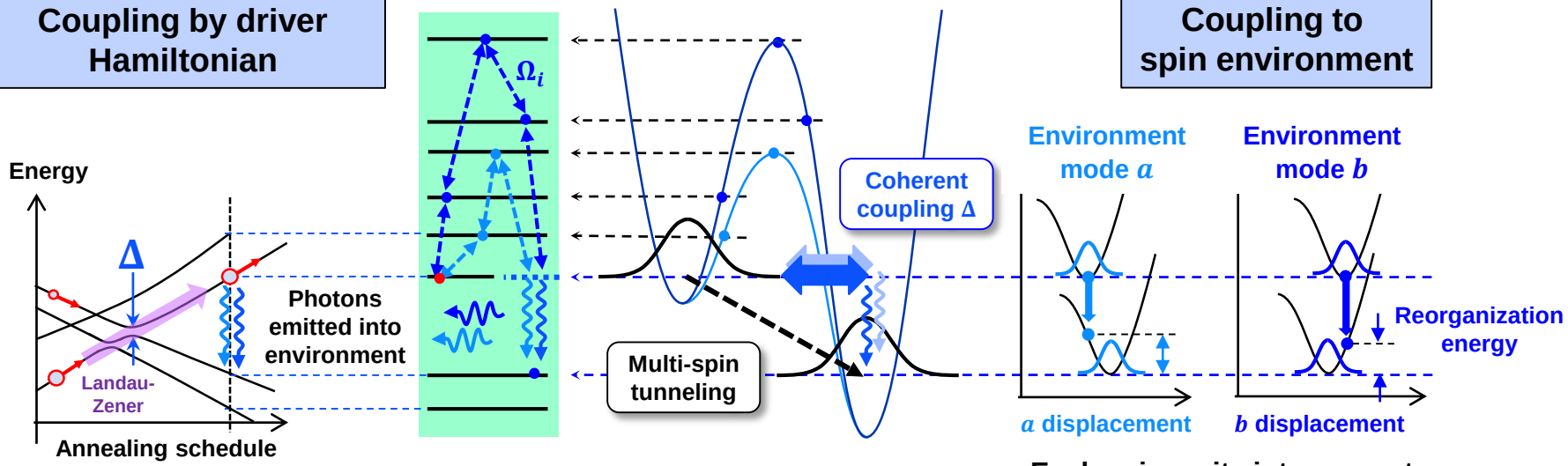
Too much coherence (reversibility) prevents thermalization

LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Increasing the "Quantum" in QA

Coupling by driver Hamiltonian

Coupling to spin environment



Each spin emits into separate modes ≡ "incoherent tunneling"

Taken together, these considerations suggest a combination of:

- **High spin coherence, either natively or with quantum error suppression**
- **Multi-spin quantum fluctuations; AND:**
- **Engineered quantum environments to re-introduce thermalization**

LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

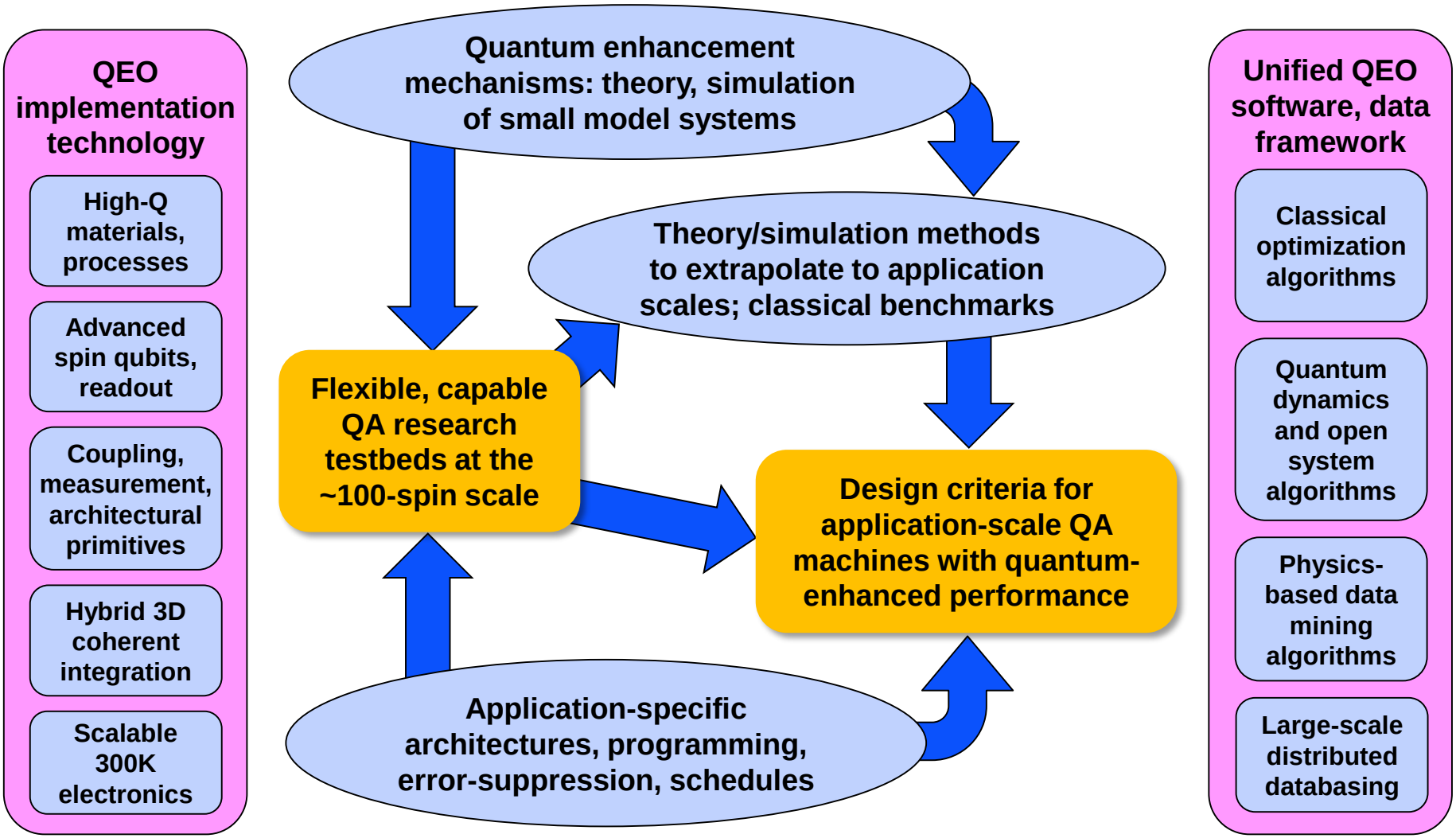
Outline

- Introduction and motivation
- **Overview of plan developed in QEO study**
- QEO study concepts and progress
  - Physical hardware and architecture concepts: this talk
  - Materials, fabrication, and experiment: Will Oliver – next talk

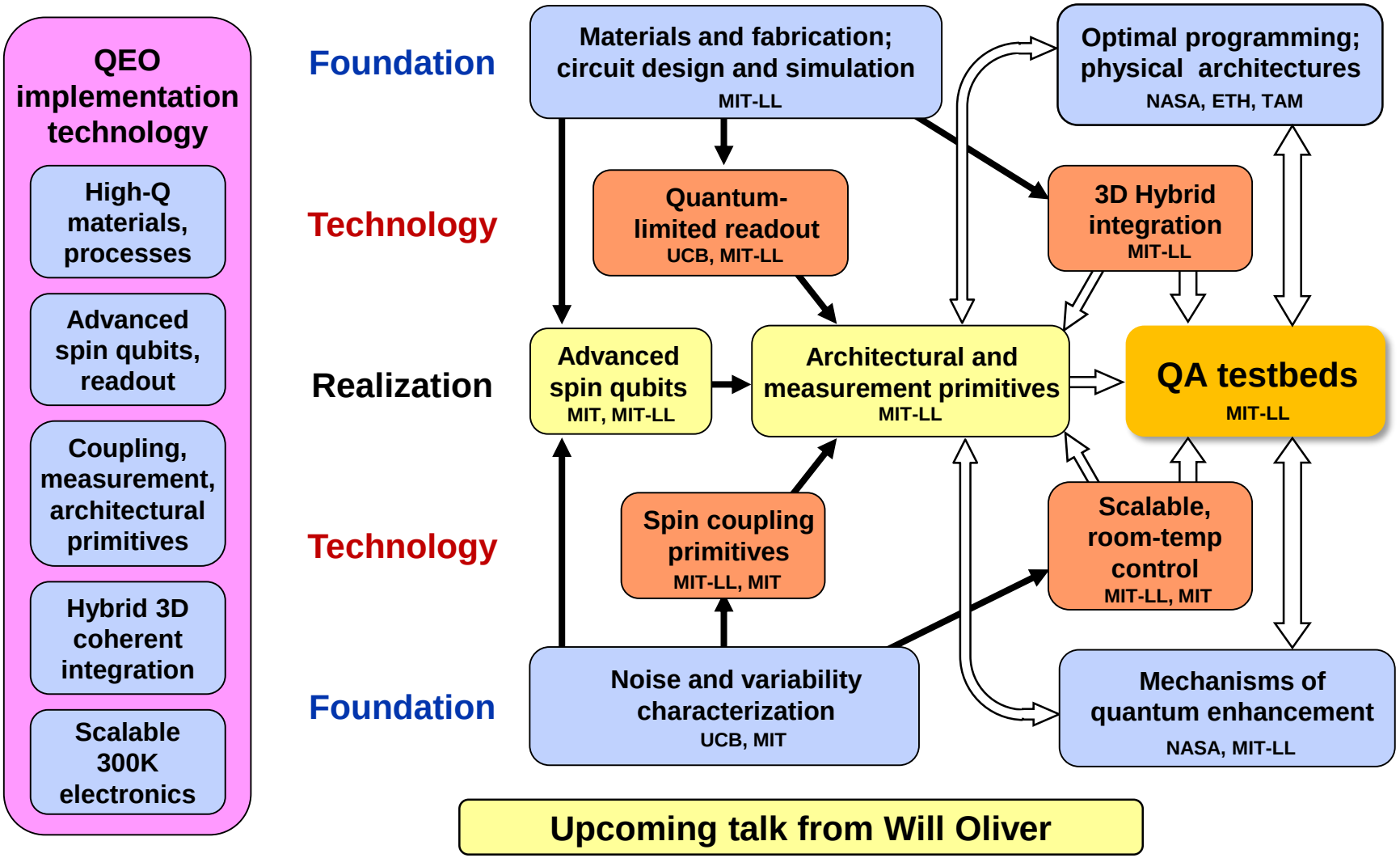
    Representing also work of MIT and UCB
  - Theory and simulation: Matthias Troyer – final talk

    Representing also work of NASA and TAMU

High-Level Plan Developed in the QEO Study



QEO Implementation Overview



LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

QEO Theory and Simulation Hierarchy

Physical, programming constraints imposed by applications

Scalability of enhancement and physical requirements

Nature of and requirements for quantum enhancement

Application problems
ETH, NASA

Many thousands of spins

- Application problems with specific mapping and embedding
- New simulation methodologies required

Engineered problems
TAM, ETH

Thousands of spins

- Problems "native" to physical architectures
- Path-integral Monte-Carlo-based methods

Tailored problems
NASA, MIT-LL

Tens of spins

- Designed from to exhibit and investigate enhancement
- Full open-system simulation

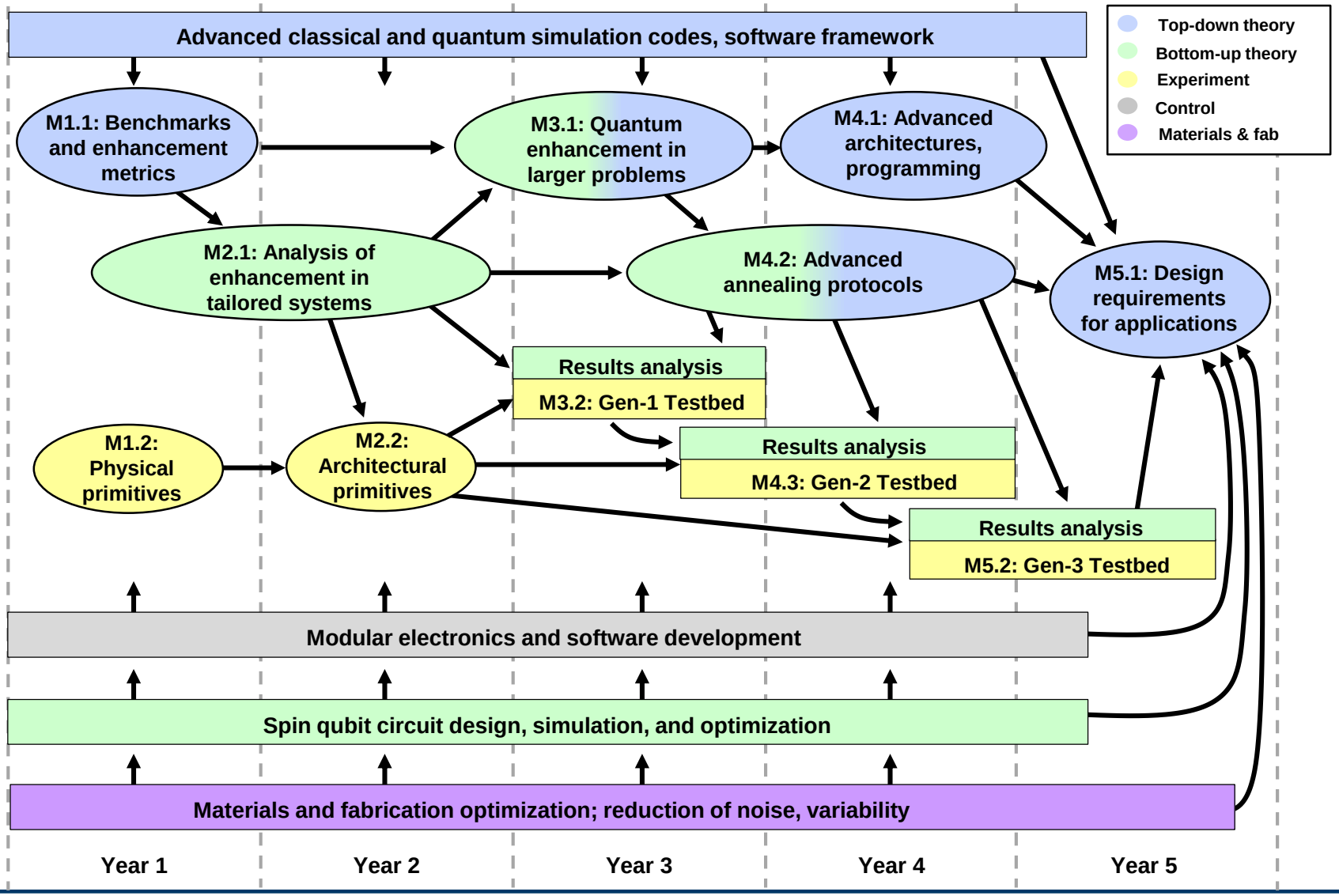
Unified QEO software, data framework

- Classical optimization algorithms
- Quantum dynamics and open system algorithms
- Physics-based data mining algorithms
- Large-scale distributed databasing

Upcoming talk from Matthias Troyer

LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Planned QEO Milestones



Outline

- Introduction and motivation
- Overview of plan developed in QEO study
- **QEO study concepts and progress**
  - **Physical hardware and architecture concepts: this talk**
  - **Materials, fabrication, and experiment: Will Oliver – next talk**

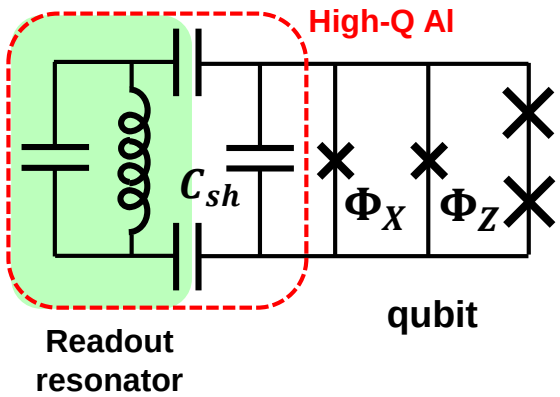
    Representing also work of MIT and UCB
  - **Theory and simulation: Matthias Troyer – final talk**

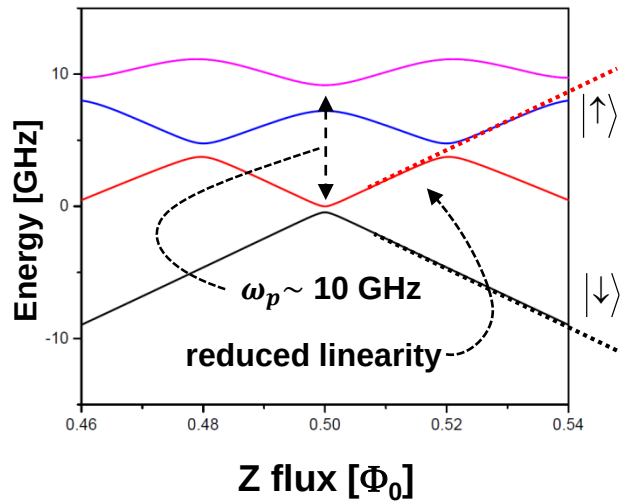
    Representing also work of NASA and TAMU

LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

MIT-LL Flux Qubits for QEO

MIT-LL C-shunt flux qubit





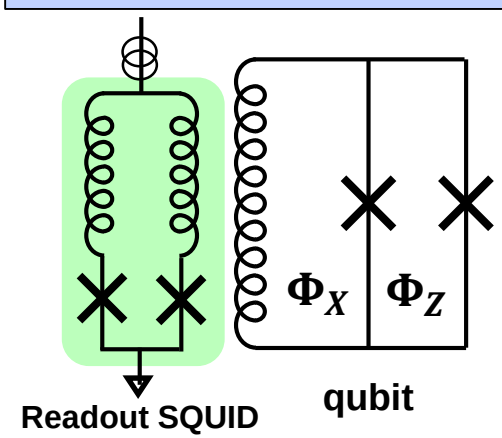
Hybrid process:
Shadow-evaporated Al JJs;
High-Q metal shunt capacitor
~ 30-50 fF

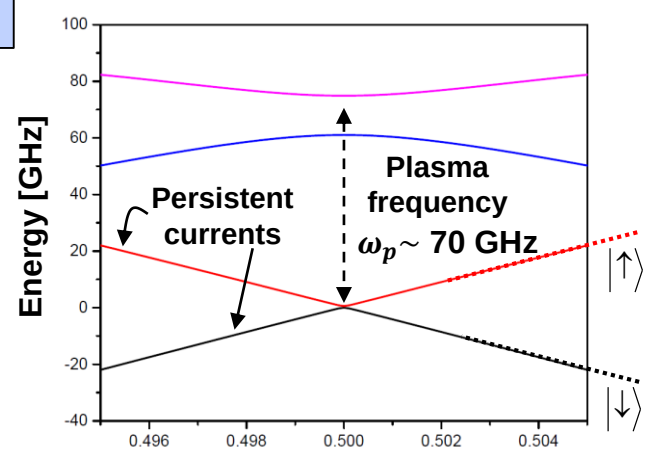
Persistent current < 0.1 μA

Coherence time up to ~50 μs
(X large, Z=0)

Capacitively-coupled, variable strength, continuous or pulsed

D-Wave RF SQUID flux qubit





Multilayer, planarized Nb process

Persistent current ~ 2 μA

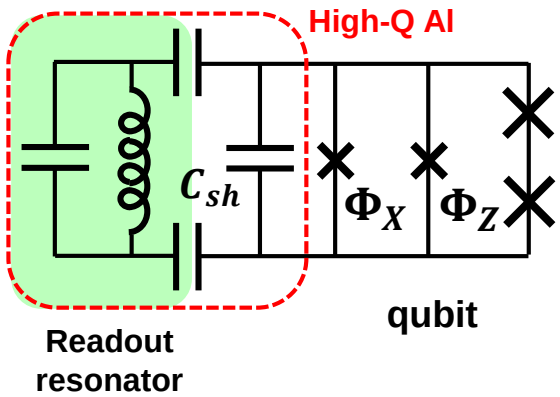
Coherence time << 1μs
(X large, Z=0)

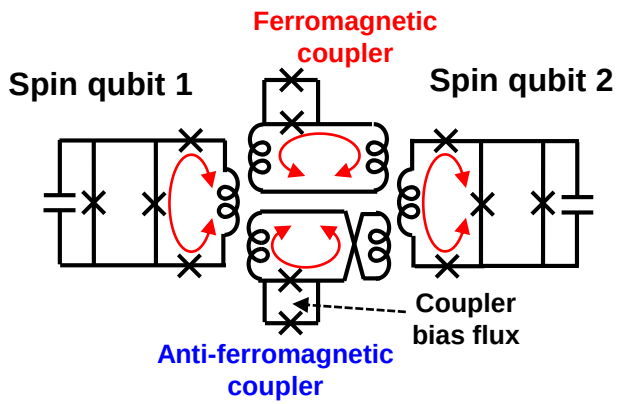
Inductively-coupled, projective readout

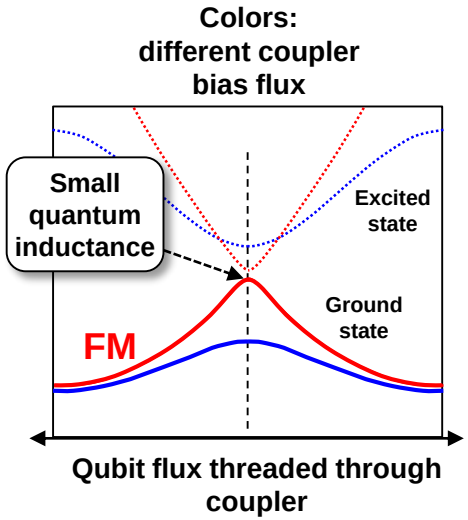
LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Basic QEO Spin Qubit Coupling

MIT-LL C-shunt flux qubit



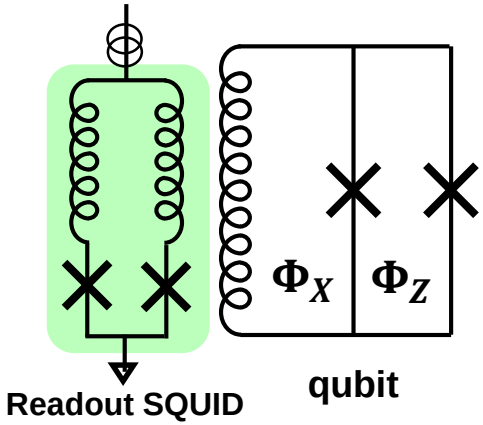


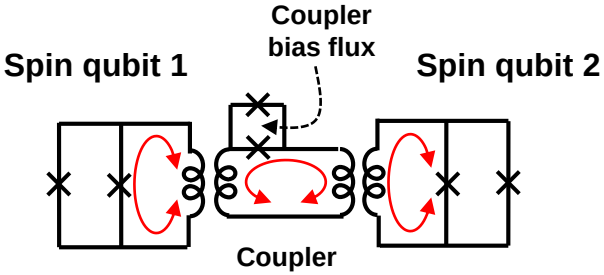


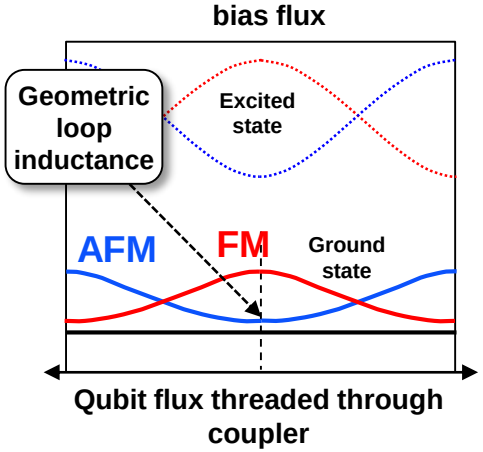
Spin coupling energy $\sim \frac{M^2 I_p^2}{L_C}$

$$L^{-1} = \frac{d^2 E}{d\Phi^2}$$

D-Wave RF SQUID flux qubit







LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

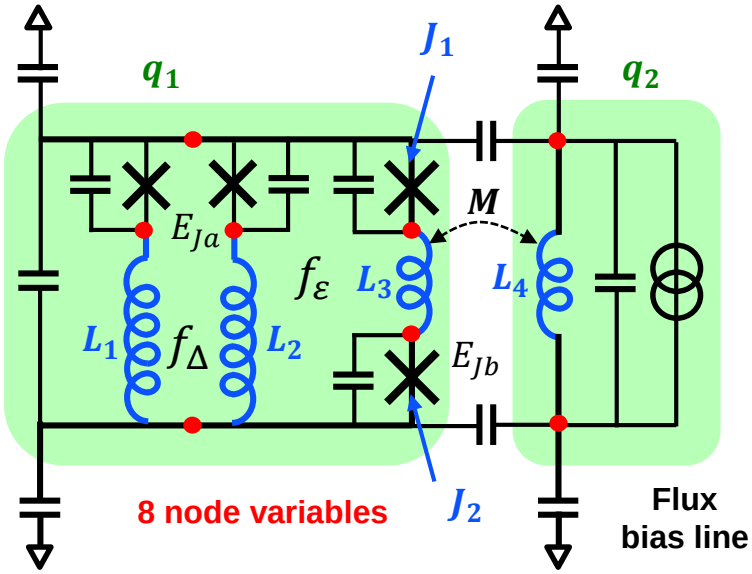
Design Simulation of Flux Qubits

Divide Hilbert space into linear oscillators, Josephson variables and island charging variables

Example - floating two-loop flux qubit with local flux bias:

$\boldsymbol{L} \in \{\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}\}$ $\quad \boldsymbol{q} \in \{\mathbf{1},\mathbf{2}\}$ $\quad \boldsymbol{J} \in \{\mathbf{1},\mathbf{2}\}$

4 linear oscillator variables + 2 free charging variables + 2 Josephson variables $= 8$



Josephson potential:

JJ charge displacement

Oscillator charge displacement

External flux biases

$$\hat{U}_J = -\frac{E_{Jb}}{2}\left(\hat{D}_{J1}^{+} + \hat{D}_{J2}^{+}\right)\hat{D}_{L3}^{+} - \frac{E_{Ja}}{2}\hat{D}_{J1}^{-}\hat{D}_{J2}^{-}\left[\left(\hat{D}_{L1}^{-}\right)^2 e^{i(f_\varepsilon + f_\Delta)} + \left(\hat{D}_{L2}^{-}\right)^2 e^{i f_\varepsilon}\right] + \text{h.c.}$$

Oscillator Hamiltonian:

$$\hat{H}_{osc} = \sum_{L=1}^{4} \hbar\omega_L\left(\hat{N}_L + \frac{1}{2}\right)$$

Inductive energy:

$$\hat{U}_M = 2\left(\frac{\Phi_0}{2\pi}\right)^2 \vec{\varphi}_L \cdot \mathbf{L}^{-1} \cdot \vec{\varphi}_L$$

Branch inductance matrix

Electrostatic energy:

$$\hat{T}_C = 2e^2 \vec{n} \cdot \mathbf{C}^{-1} \cdot \vec{n}$$

Node capacitance matrix

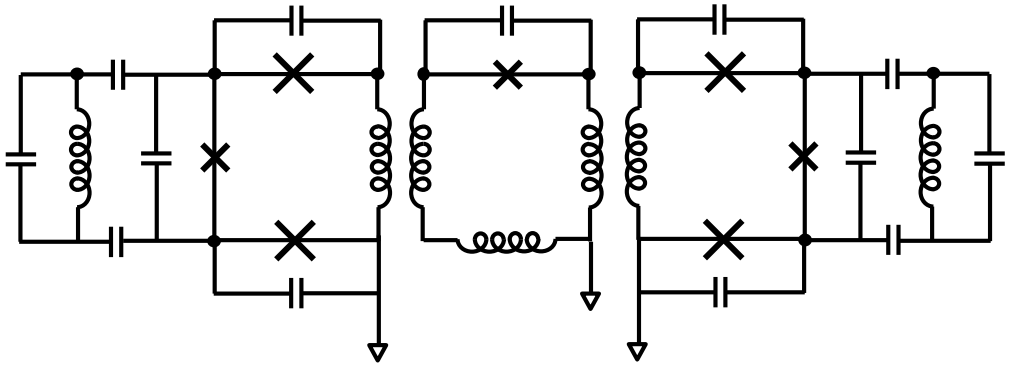
Free charging Hamiltonian:

$$\hat{H}_q = \sum_{q=1}^{2} \frac{2e^2}{C_q}\left(\hat{n}_q\right)^2$$

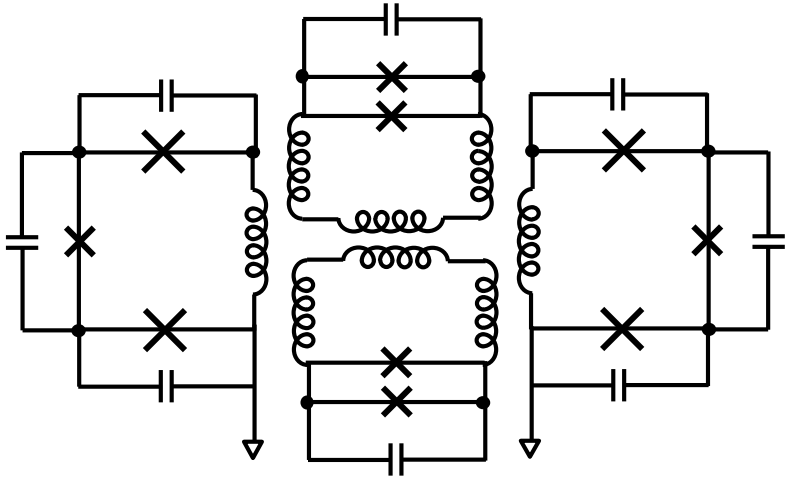
LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

More Complex Examples

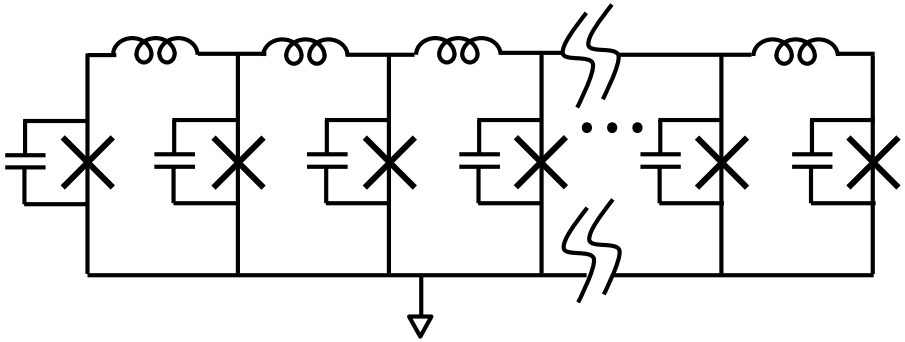
Two coupled qubits with two readout resonators



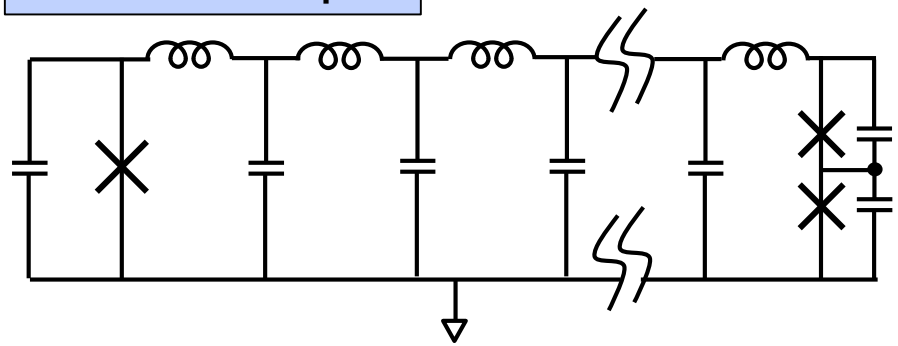
Push-pull coupling of two qubits



Multiloop SQUID



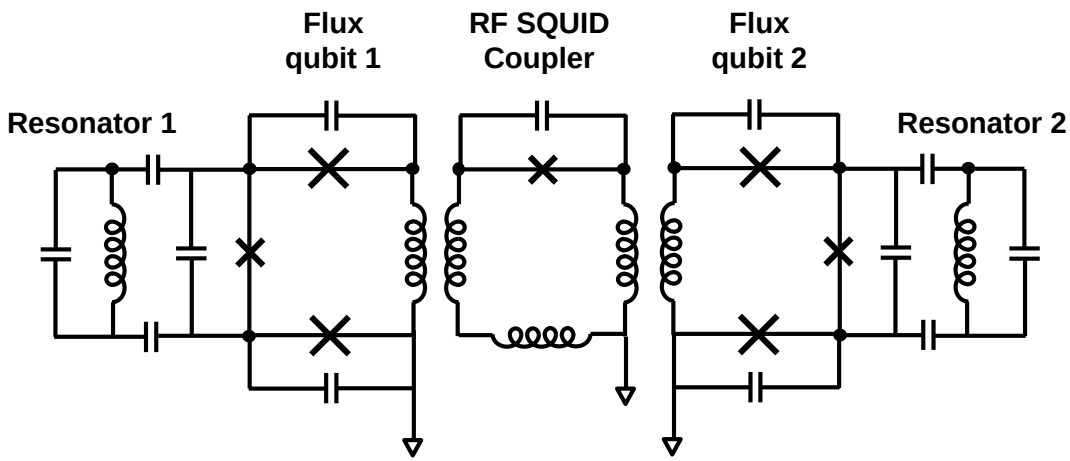
Distributed flux qubit



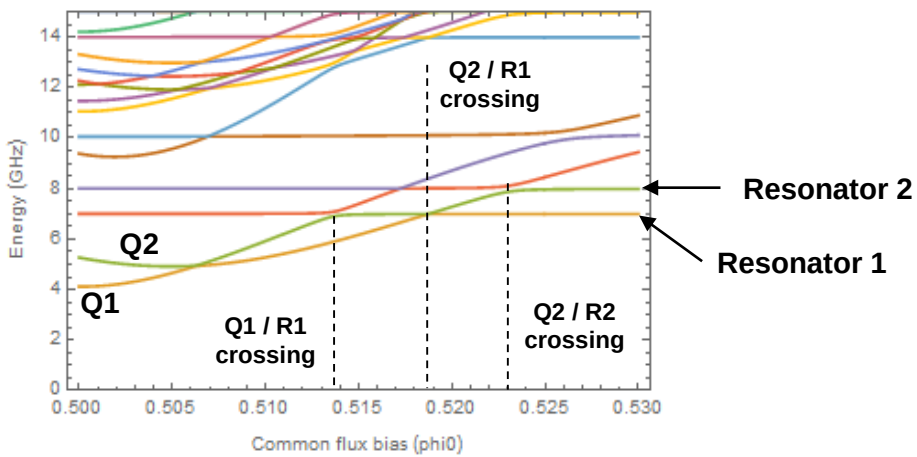
Hierarchical simulation approach allows compact description of complex circuits

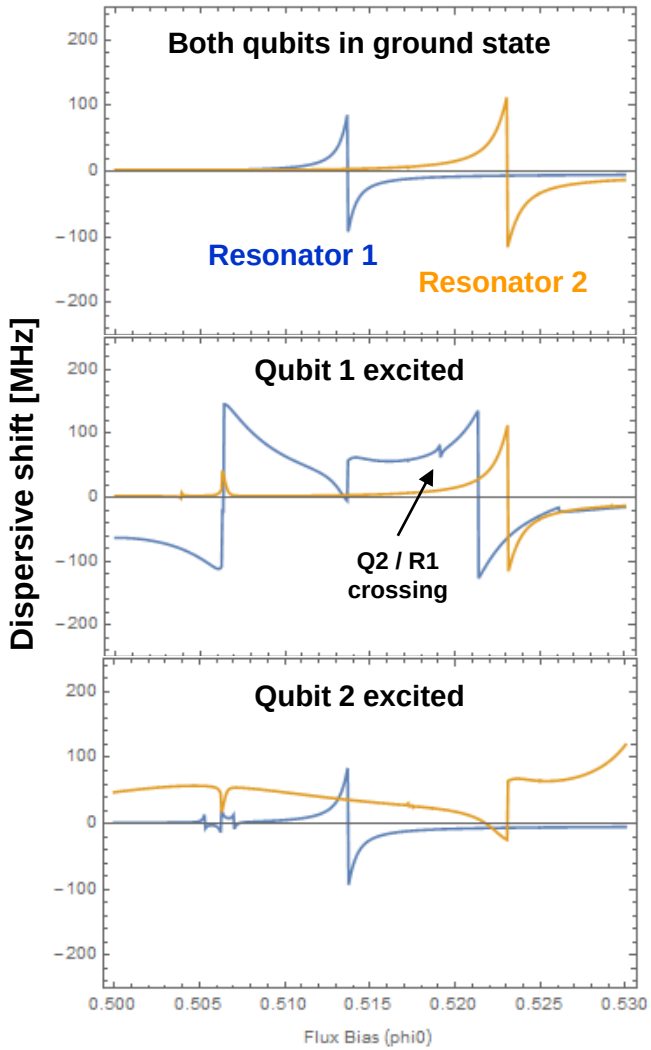
LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Example: Coupled Flux Qubits with Two Readout Resonators



Dressed states for 2 coupled qubits, 2 resonators





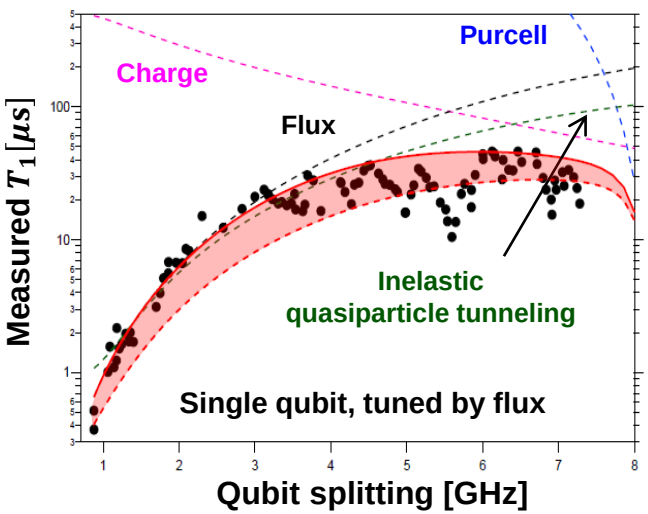
LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

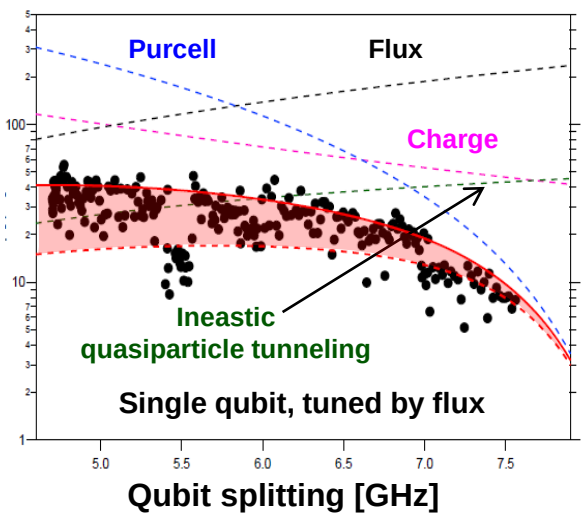
Simulation-Based Coherence Model

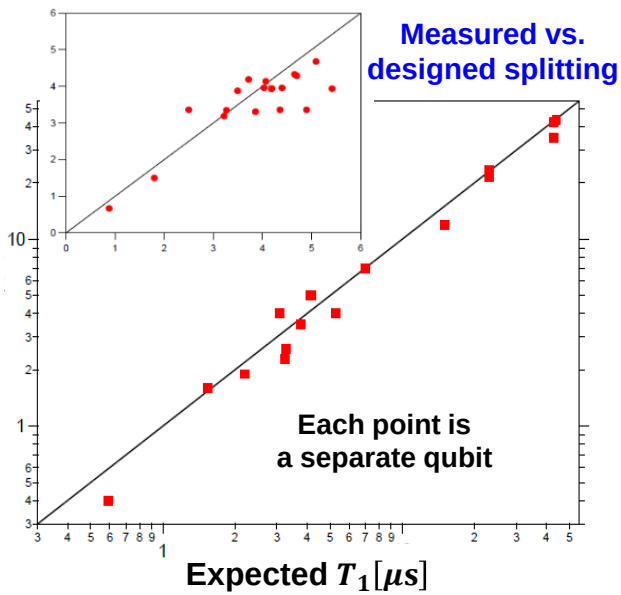
Magnetic noise | **Electric noise** | **Quasiparticle shot noise** | **Purcell effect**

Loop current — Loop flux noise — Node voltage — Node charge noise — QP tunneling — QP current noise — Resonator linewidth — Resonator position

$$\frac{1}{T_1} = \frac{1}{\hbar^2}\left[\sum_i \underbrace{|\langle g|\hat{I}_i|e\rangle|^2}_{\text{Magnetic dipole} \sim E_J} S_{\Phi i} + \sum_j \underbrace{|\langle g|\hat{V}_j|e\rangle|^2}_{\text{Electric dipole} \sim E_C} S_{Qj} + \sum_k \underbrace{|\langle g|\hat{T}_k|e\rangle|^2}_{\text{QP tunneling dipole} \sim E_J} S_e\right] + \frac{\kappa}{2}\underbrace{|\langle g;1|\hat{x}_r|e;0\rangle|^2}_{\text{Dressed qubit states}}$$



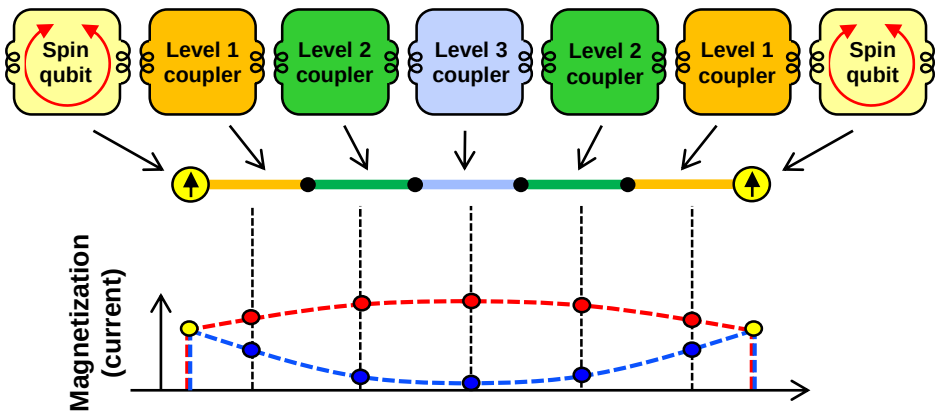




LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Two-Spin Coupling Primitives

Two-spin coupling "primitive"

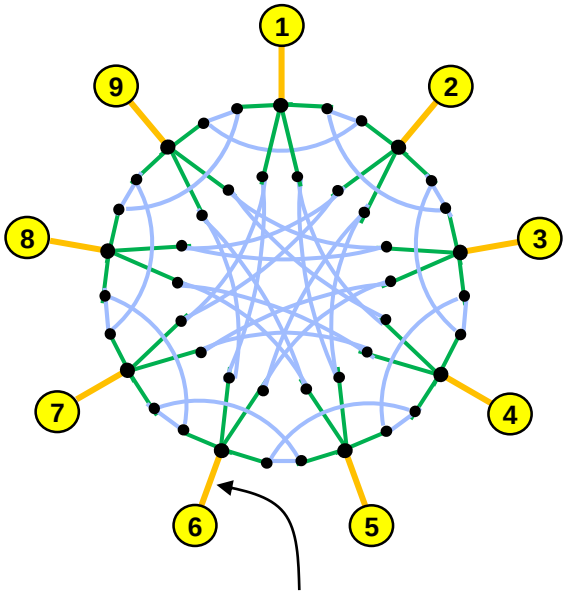


Spins are field sources in an effective paramagnetic medium

Independent X-Z coupling angle for each spin

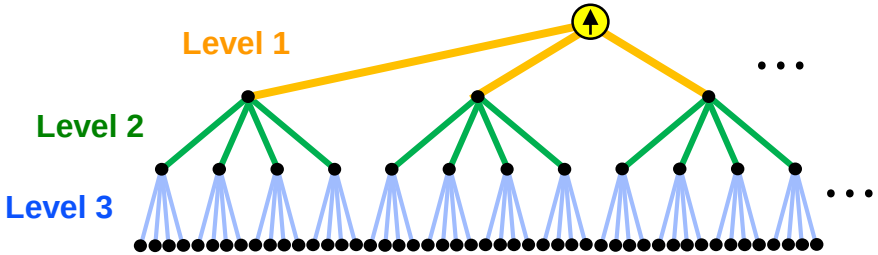
$$\hat{H}_{eff} = J\left[\hat{\sigma}_1^z \cos(\beta_1) + \hat{\sigma}_1^x \sin(\beta_1)\right] \otimes \left[\hat{\sigma}_2^z \cos(\beta_2) + \hat{\sigma}_2^x \sin(\beta_2)\right]$$

Example: 9-spin fully-connected cluster



Only one direct inductive coupling to each spin qubit

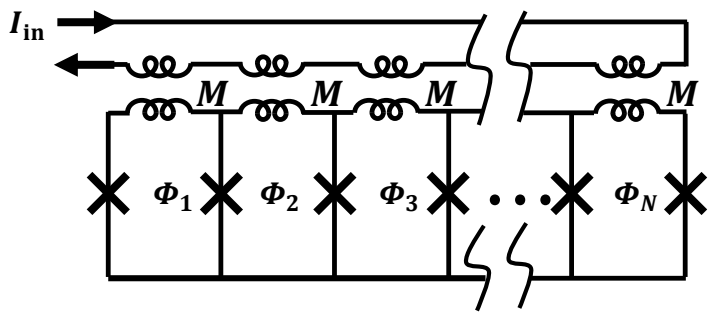
Each level 3 coupler controls an independent J_{ij}



LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

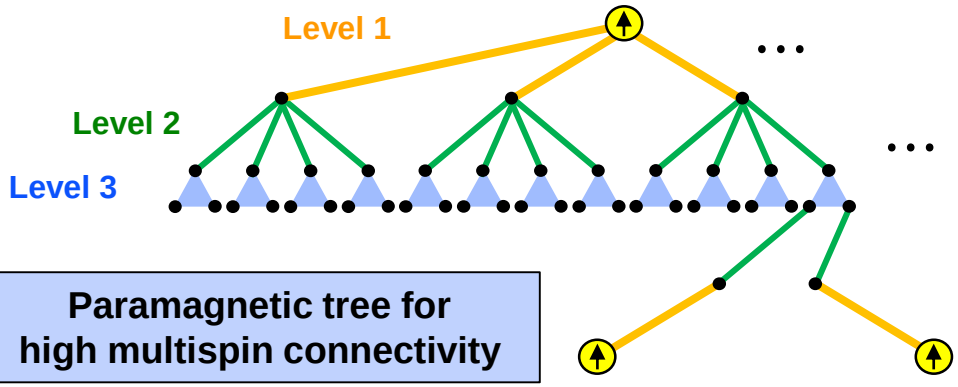
Multi-Spin Coupling Primitives

Flux-coupled multiloop SQUID interferometer

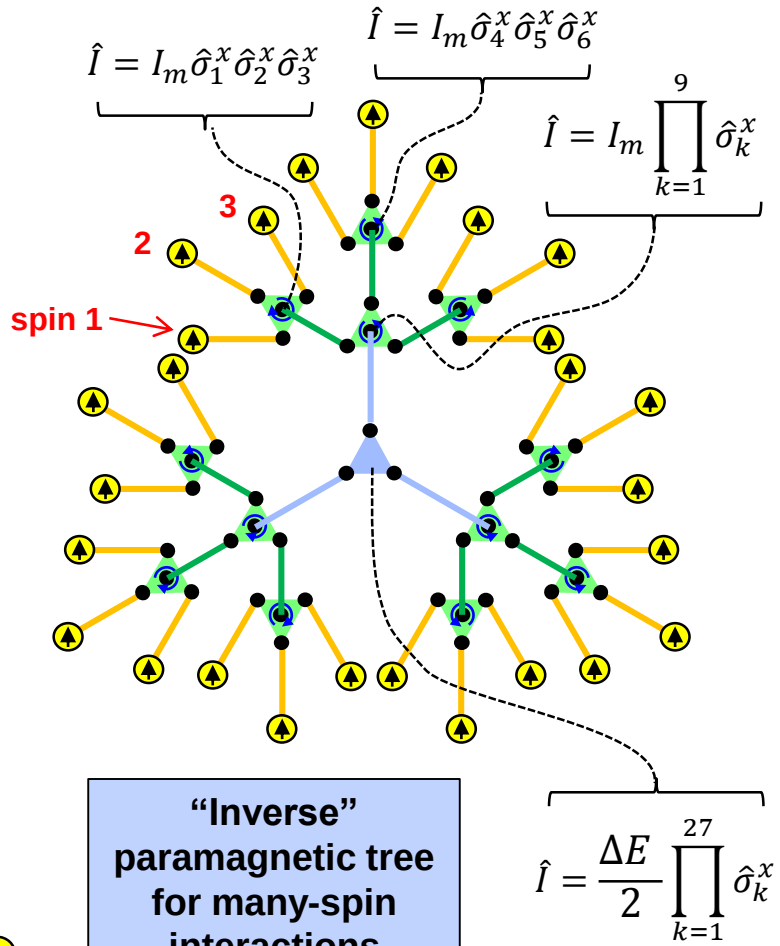


$$\hat{H}_3 = \frac{\Delta E}{2}\hat{\sigma}_1^z\hat{\sigma}_2^z\hat{\sigma}_3^z$$

$$\hat{I}_3 = I_m\hat{\sigma}_1^z\hat{\sigma}_2^z\hat{\sigma}_3^z$$



Paramagnetic tree for high multispin connectivity



"Inverse" paramagnetic tree for many-spin interactions

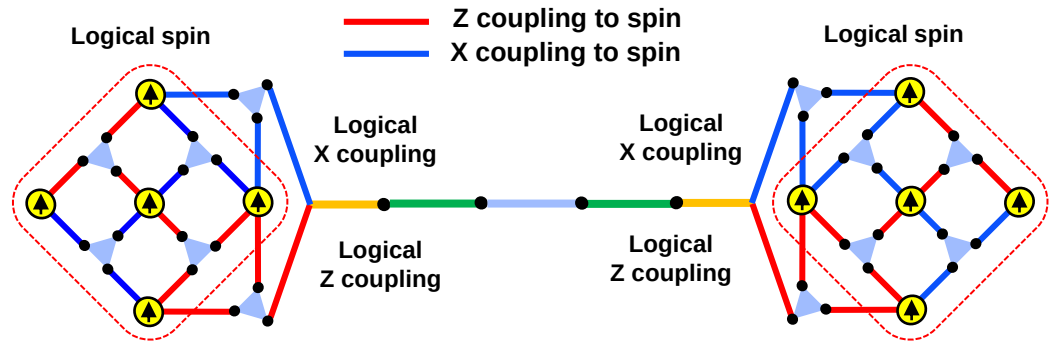
LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Applications of Multi-Spin Interactions

$$\hat{H}_3 = \sum_{i<j<k} \left\{ J^Z_{ijk} \hat{\sigma}^z_i \hat{\sigma}^z_j \hat{\sigma}^z_k + J^X_{ijk} \hat{\sigma}^x_i \hat{\sigma}^x_j \hat{\sigma}^x_k \right\}$$

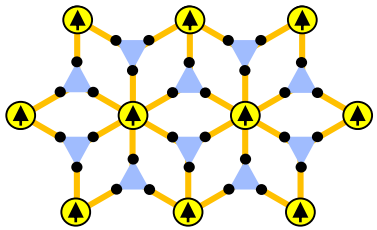
3-spin Ising interaction

3-spin fluctuations

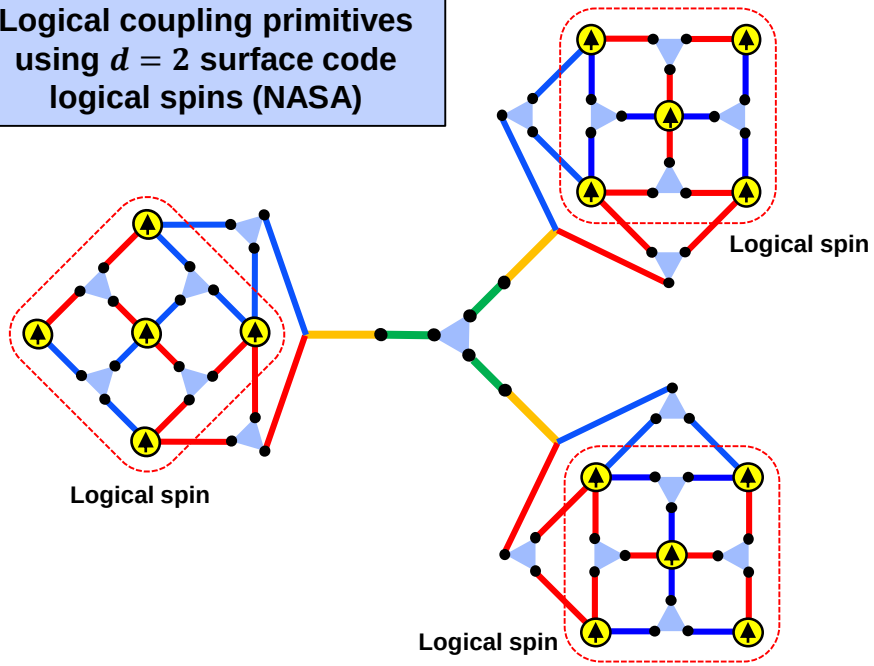


Ising model on *planar* hypergraph is NP hard for $k = 3$

C. Thomas, H. Katzgraber, PRE 83 046709 (2011)

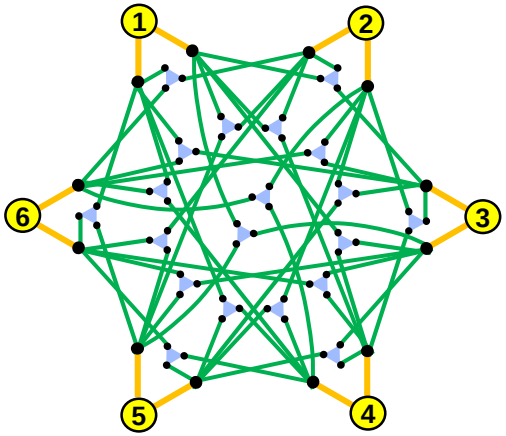


Logical coupling primitives using $d = 2$ surface code logical spins (NASA)



Multi-spin fluctuations exponentially more efficient

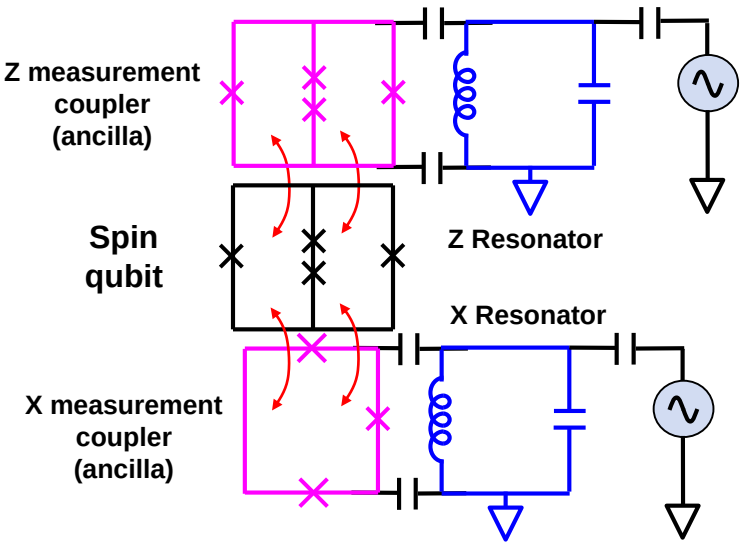
$k = 3$ fully-connected cluster

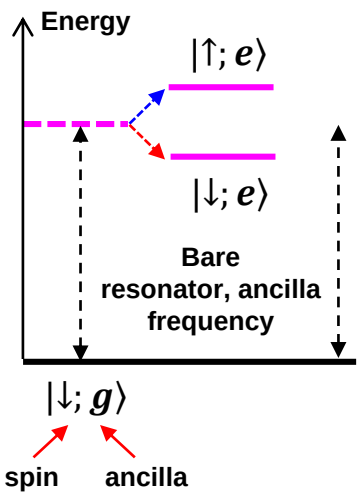


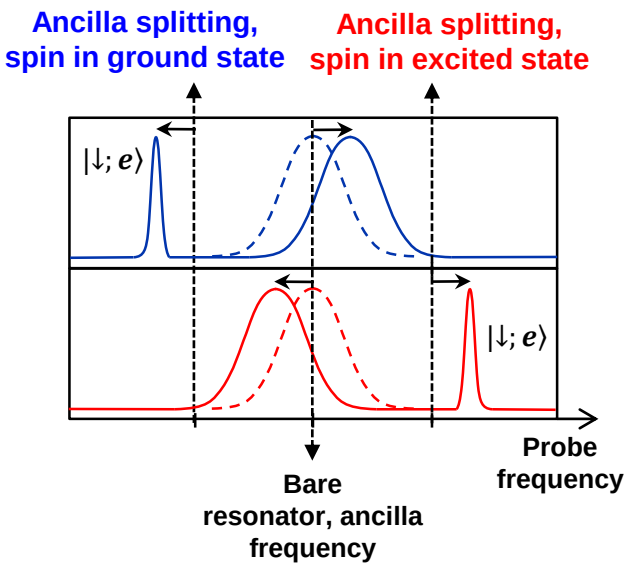
LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Spin Measurements

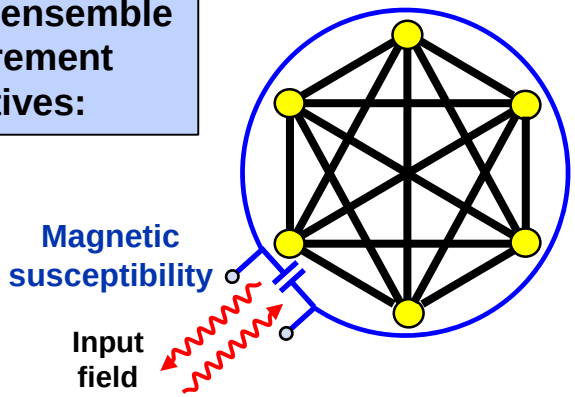
Tunable-strength vector spin measurement:

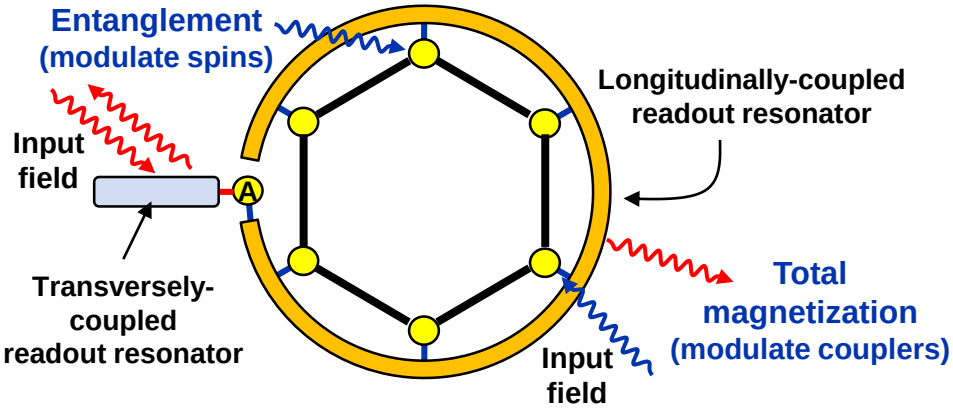






Candidate ensemble measurement primitives:





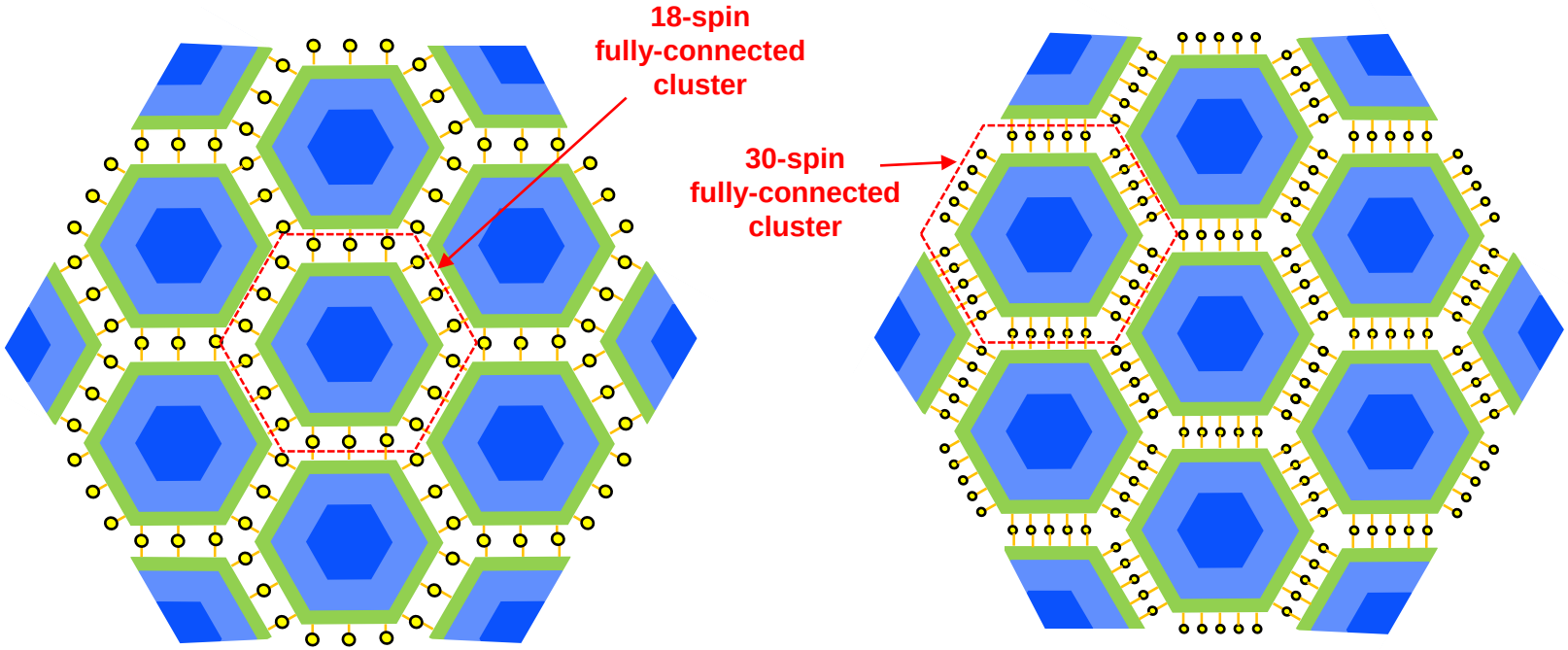
LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Example Testbed Architecture (Two-Spin Couplings)

90 spin qubits
connectivity: 32

150 spin qubits
connectivity: 54

"Architectural primitives"

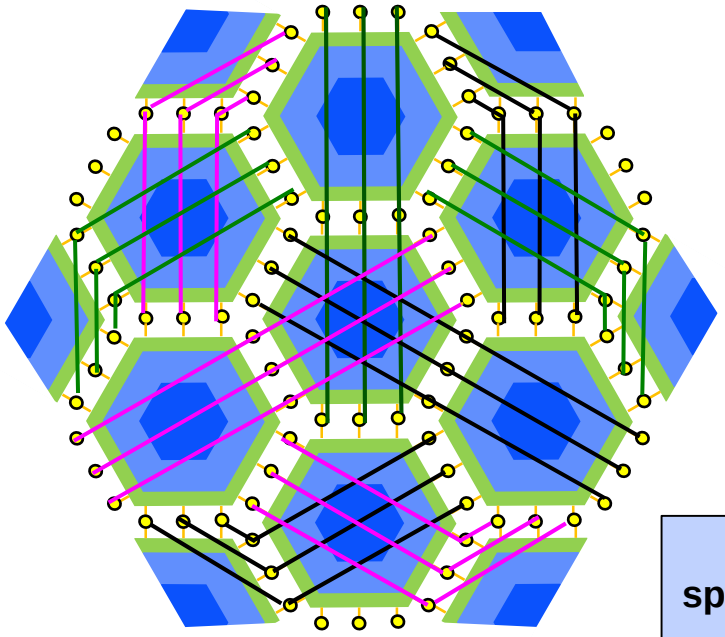


2268 two-spin couplers;
minimum 1458 active

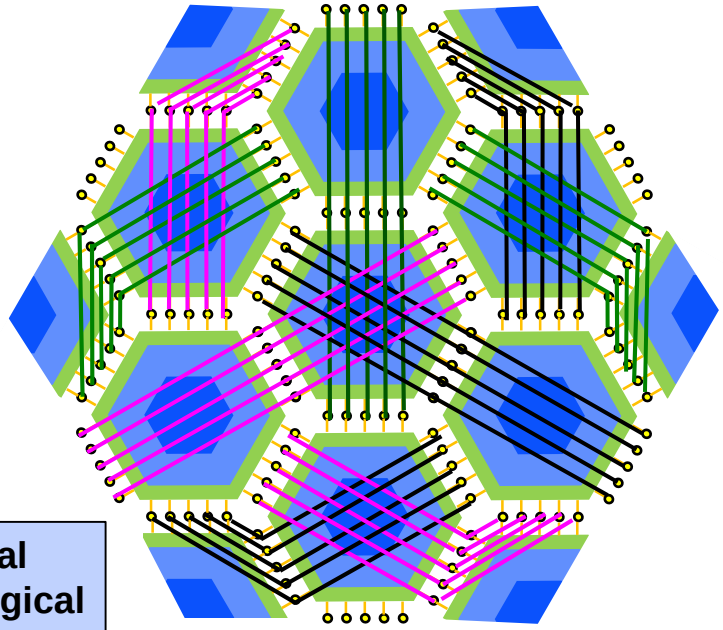
5670 two-spin couplers;
minimum 4050 active

Example Testbed Architecture (Two-Spin Couplings)

90 spin qubits
connectivity: 32



150 spin qubits
connectivity: 54

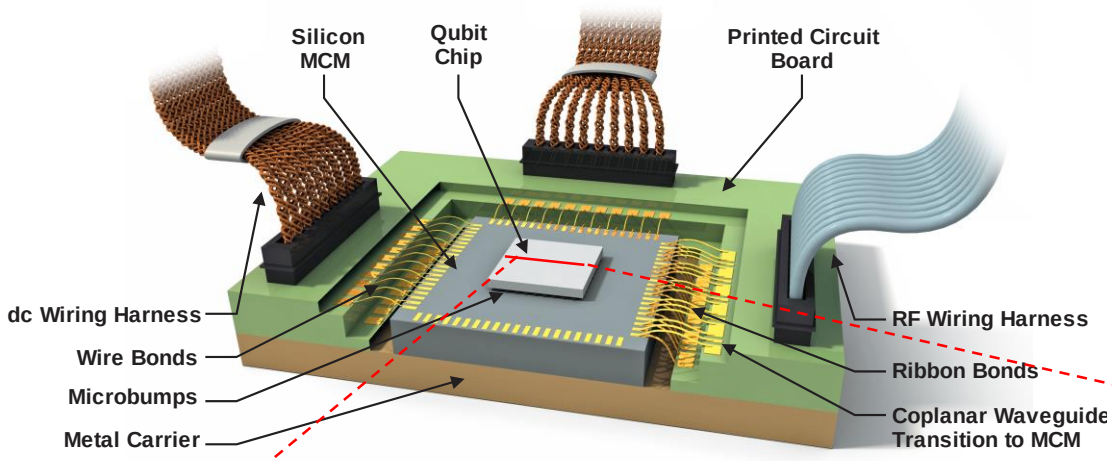


3 physical spins per logical spin

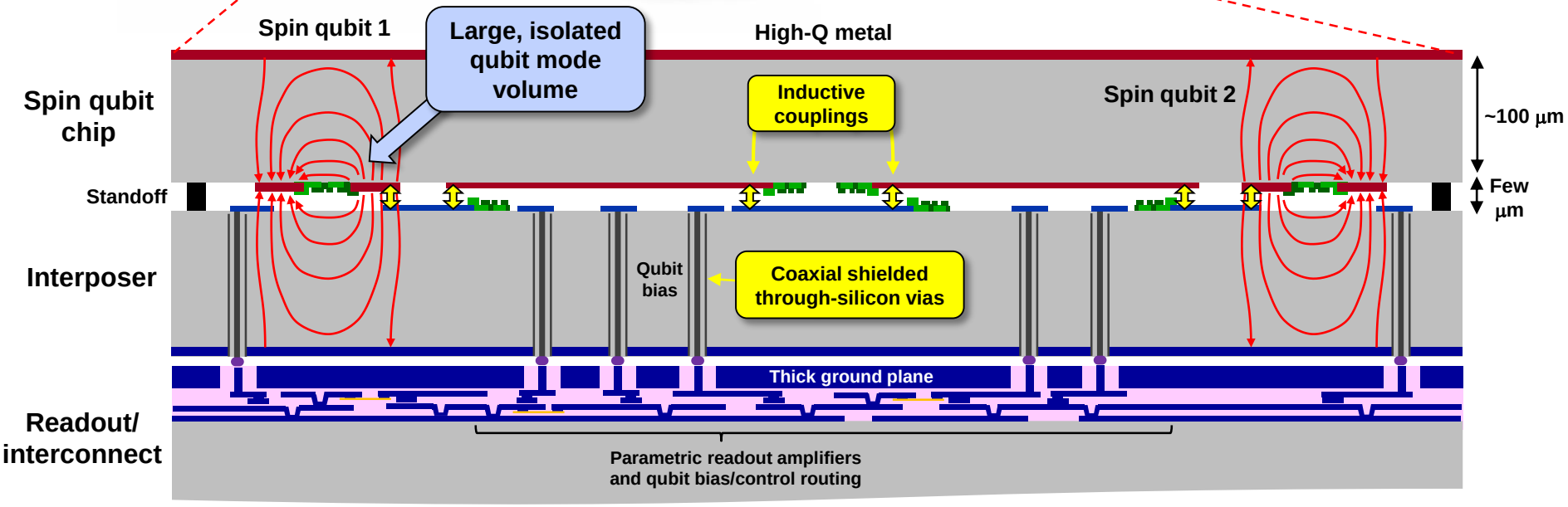
27 logical spin
fully connected cluster embedding

45 logical spin
fully connected cluster embedding

Quantum Annealing Testbed Concept from QEO Study



- High interconnect density allows highly-capable, reconfigurable room-temp electronics
- Qubits, interconnects, control are optimized separately, independently
- Maximize isolation of qubits from lossy and uncontrolled surfaces



LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Summary and Conclusion

Goals of QEO:

Determine whether QA can provide optimization performance beyond classical methods for specific problems

Detailed requirements for realizing this at application scales, for specific problems

Study achievements

- **A comprehensive plan to achieve these goals**
  - **Firmly grounded in theory**
  - **Explores novel capabilities beyond that of commercial systems, while minimizing risk:**
    - **Isolates (potentially conflicting) technology development efforts**
    - **Defers ancillary technology development wherever possible**
- **Excellent progress in executing this plan: a firm foundation to build on**
  - **High coherence materials/fab, QA-optimized flux qubits, readouts, and integration**
  - **Advanced physical architecture concepts**
  - **Major theoretical advances towards ultimate goal of projecting large-scale performance**

Major step towards establishing plausibility of disruptive QA capabilities